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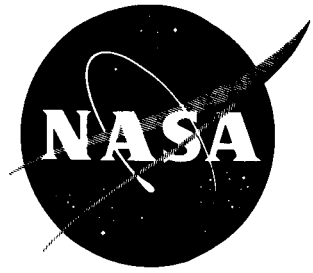

TECHNICAL NOTE

D-80

DEPARTURE TRAJECTORIES FOR INTERPLANETARY VEHICLES

By W. E. Moeckel

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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DEPARTURE TRAJECTORIES FOR INTERPLANETARY VEHICLES

By W. E. Moeckel

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SUMMARY

General expressions are derived for the magnitude and direction of the launching velocity needed to reach another planet from any point on the Earth's surface. The effect of the inclination of the orbital plane of the destination planet is considered, as well as the effect of departure date and selected trajectory. A procedure is given for determining optimum time of day for launching from any latitude to obtain maximum benefit of the Earth's rotation. Results of the analysis are illustrated for typical Earth-Venus trajectories.

The analysis indicates that launching of interplanetary vehicles can take place from any latitude with velocity penalties no greater than loss of the benefit of the Earth's rotation. The best launching latitude is found to depend strongly on the trajectory to be followed and is not necessarily the equator. By proper selection of time of day, most of the benefit of the rotational component of the Earth at the launch latitude can be realized. Penalties associated with launching from satellite orbits can be much more severe, amounting to loss of the benefit of the orbital speed of the vehicle, if the orbit plane is improperly inclined.

INTRODUCTION

The problem of determining the correct magnitude and direction of the launching velocity required to reach another planet from a given point on the Earth's surface is a fairly complicated one. An exact solution requires numerical integration of the multibody trajectory equations, but these are feasible only when the initial conditions have been narrowed to a limited range by means of approximate analyses. These approximate analyses consist of application of the equations of motion of a small mass in the gravitational field of a single, much larger mass.

For the case of interplanetary flights, considerable work has been done on the heliocentric trajectories suitable for transfer from the orbit of the Earth to the orbit of other planets (refs. 1 to 3). These

calculations have assumed that the orbit of the destination planet is coplanar with the Earth's orbital plane. In reference 1, the one-way trajectories are given in very general form in terms of the initial heliocentric velocity vector of the vehicle. Several families of round-trip trajectories are presented in reference 2 for the Mars trip and in reference 3 for the Mars and Venus trips. Total trip times are given as functions of the magnitude of the velocity increments produced by the propulsion system and of the magnitude of the hyperbolic velocities at the Earth and at the destination planet. (The hyperbolic velocity is the velocity of the vehicle relative to the departure or destination planet at large distances from the planet.) These analyses provide a basis for picking desirable transfer trajectories, but they do not consider three-dimensional problems such as effect of inclination of the orbital planes of the departure and destination planets or determination of the magnitude and direction of the launch velocities required at particular points on the planet's surface to achieve the desired trajectory. A general discussion of the three-dimensional problems occurring when departure starts from a satellite orbit is presented in reference 4, wherein it was shown that large velocity penalties, relative to two-dimensional values, can result if the satellite orbit plane differs considerably from the required launching plane.

In the present report, these three-dimensional problems are considered from the standpoint of launching interplanetary vehicles from planet surfaces. Expressions are first derived for the effect of inclination of the orbital plane of the destination planet on the magnitude and direction of the hyperbolic velocity vector. The direction and magnitude of the burnout velocity required to obtain these hyperbolic velocities are then determined as functions of launching latitude and launching time. Results of the analyses are illustrated for typical Earth-Venus trajectories.

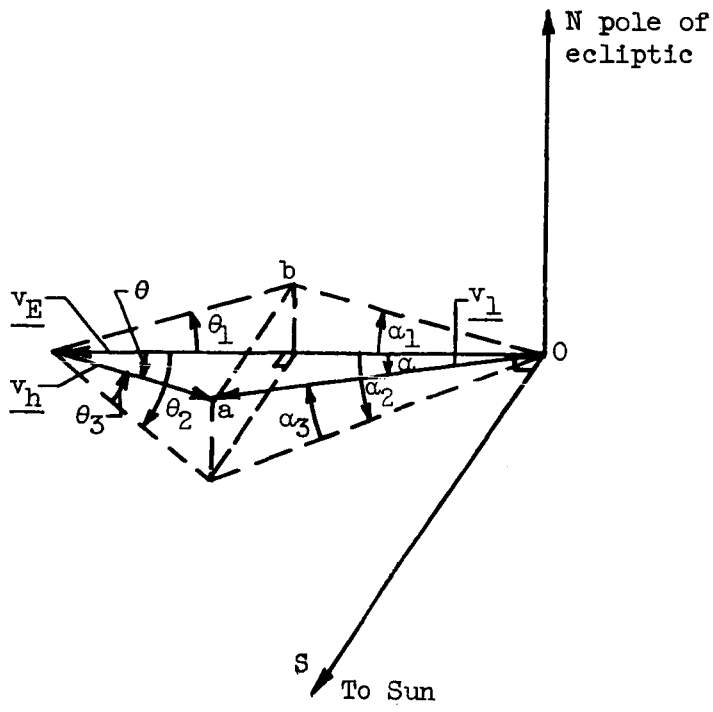
ANALYSIS

The analysis is presented in two main phases: First, the relation between the required hyperbolic velocity and the desired initial heliocentric velocity is determined with the effect of the inclination of the orbital plane of the destination planet considered. Second, the magnitude and direction of the launch velocity needed to attain the required hyperbolic velocities are derived.

For convenience, the derivations are presented in terms of a launch from the Earth to another planet. With appropriate changes in reference system, however, the equations are equally applicable to launching from another planet. Also, the use of hyperbolic velocity is not essential to the analysis. In place of this velocity, for example, the required

velocity at the boundary of a sphere of influence of the launching planet could be used. Such a substitution is desirable in many applications since there is a definite distance associated with such a velocity, whereas the hyperbolic velocity is exactly attained only at infinite distance from the planet and in the absence of other gravitational fields.

Relations Between Heliocentric and Hyperbolic Velocity



(a)

Shown in sketch (a) is the general relation between the heliocentric velocity $\underline{v_1}$ required at the Earth's orbit to reach the destination along a specified coplanar trajectory and the hyperbolic velocity $\underline{v_h}$ required to attain this value of $\underline{v_1}$. (Underlined quantities signify vectors in this report, and symbols are defined in the appendix.) The lines to the Sun and the Earth's orbital velocity vector $\underline{v_E}$ determine the ecliptic plane. The Earth's orbit is assumed to be circular; hence, $\underline{v_E}$ is perpendicular to the Sun-Earth line. The angles α and θ are the angles of the heliocentric velocity and the hyperbolic velocity relative to the Earth's orbital velocity $\underline{v_E}$. If no midcourse corrections are to be imposed, the velocity $\underline{v_1}$ must lie in the plane

containing the Sun, the Earth at departure, and the destination planet on arrival. The velocity $\underline{v_1}$ is, therefore, the initial velocity required to follow a prescribed coplanar trajectory from the Earth's orbit to the orbit of the destination planet. Its magnitude and its component angle α_2 relative to the plane normal to O-S can be selected by coplanar methods such as those of references 1 to 3. (The angle determined from coplanar solutions is actually $\overline{aOb}$ in sketch (a), but the difference between this angle and α_2 is negligible for planetary orbit inclinations.)

With v_1 and α_2 known, α_1 can be determined in terms of the inclination of the orbital plane of the destination planet relative to the ecliptic. After α_1 is determined, the magnitude and directions of the hyperbolic velocity, as well as the other angles of sketch (a), can be calculated from the following equations:

$$\cos^2\alpha = \frac{\cos^2\alpha_1 \cos^2\alpha_2}{\cos^2\alpha_1 + \cos^2\alpha_2 - \cos^2\alpha_1 \cos^2\alpha_2} \tag{1}$$

$$v_h^2 = (v_E - v_1 \cos\alpha)^2 + v_1^2 \sin^2\alpha \tag{2}$$

$$\sin\theta = \frac{v_1}{v_h} \sin\alpha \tag{3}$$

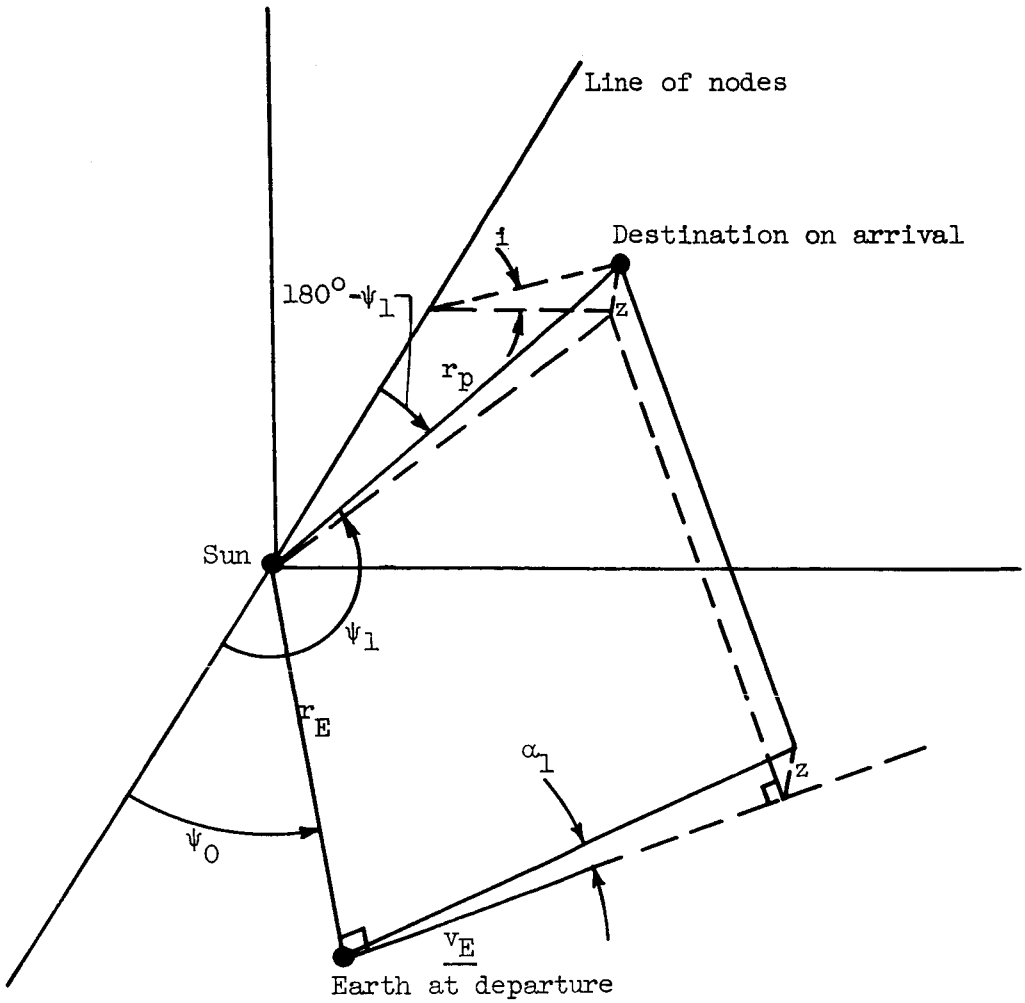
$$\tan\theta_1 = \frac{v_1 \cos\alpha}{v_h \cos\theta} \tan\alpha_1 \tag{4}$$

$$\tan\theta_2 = \frac{v_1 \cos\alpha}{v_h \cos\theta} \tan\alpha_2 \tag{5}$$

$$\cos\theta_3 = \frac{\cos\theta}{\cos\theta_2} \tag{6}$$

$$\cos\alpha_3 = \frac{\cos\alpha}{\cos\alpha_2} \tag{7}$$

The angle α_1, which is the inclination to the ecliptic of the plane containing the Sun, the Earth at departure, and the destination at arrival, can be determined as a function of time with the aid of sketch (b). Let ψ_0 be the angular distance of the Earth at departure from the ascending node of the destination planet and ψ_1 the angular distance of the



(b)

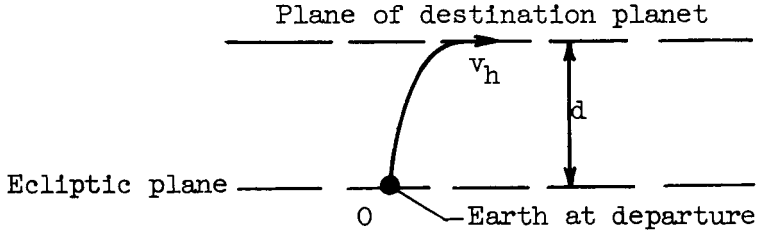
destination planet from its ascending node upon arrival of the vehicle, both angles being measured in the direction of motion of the planets. The angle ψ_0 is in the ecliptic plane, and ψ_1 is in the orbital plane of the destination. The angle i is the inclination of the orbital plane of the destination planet. Some trigonometric manipulation shows that

$$\tan \alpha_1 = \frac{\sin i \sin \psi_1}{\cos \psi_0 \sin \psi_1 \cos i - \sin \psi_0 \cos \psi_1} \approx \frac{\sin i \sin \psi_1}{\sin(\psi_1 - \psi_0)} \qquad (8)$$

Since $\cos i \approx 1$, this relation shows, as might be expected, that α_1 is approximately 90° if $\psi_1 - \psi_0 \approx 180^\circ$ except when $\sin \psi_1 = 0$. In other words, when departure and destination points are 180° apart and the

destination is not at the nodal line upon arrival, the only plane containing both of these points and the Sun is perpendicular to the ecliptic. This trajectory, of course, is prohibitively expensive in terms of velocity required, since the orbital motion of the Earth must be completely canceled and a velocity v_1 perpendicular to the Earth's orbit must be provided. Thus, the 180° trajectory (Hohmann ellipse) is only rarely a minimum-energy trajectory when the inclination of orbital planes is considered. Lower initial velocities will be required if launching takes place some days before or after the coplanar minimum-energy launching date.

Other methods of reaching the destination generally require that at least one velocity increment be provided after initial burnout. One such method is to launch directly into the orbital plane of the destination planet. This method is illustrated in sketch (c) and may require fairly high total velocity increment unless the Earth is very close to a node.



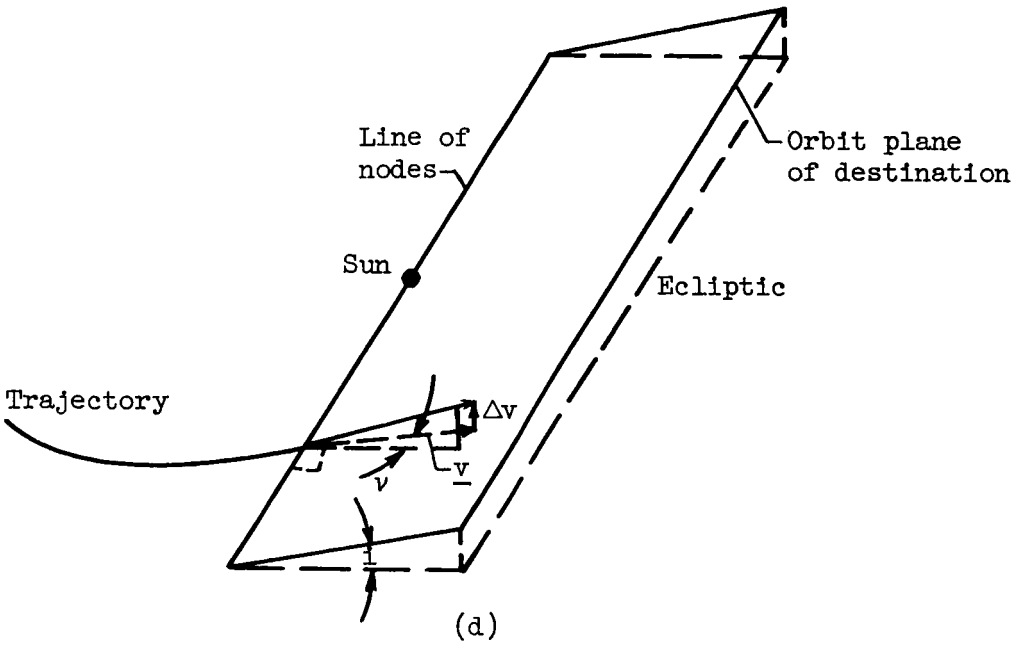
(c)

The distance between the ecliptic plane and the plane of the destination planet at launch is given by

$$d = r_E \sin i \sin \psi_0 \tag{9}$$

where r_E is the distance from Earth to Sun. Unless $\sin \psi_0 \approx 0$, d will be sufficiently large that the launch velocity to reach d must be very close to escape velocity. The required hyperbolic velocity must then be provided with an additional application of thrust when the distance d is reached.

If midcourse corrective thrust is provided, many other possibilities exist. For example, if the departure and arrival points are on opposite sides of the nodal line, an impulse can be provided - when the vehicle reaches the nodal line - to transfer the trajectory into the orbit plane of the destination planet. The impulse needed depends on the angle and velocity with which the vehicle approaches the nodal line (sketch (d)).



(d)

Thus,

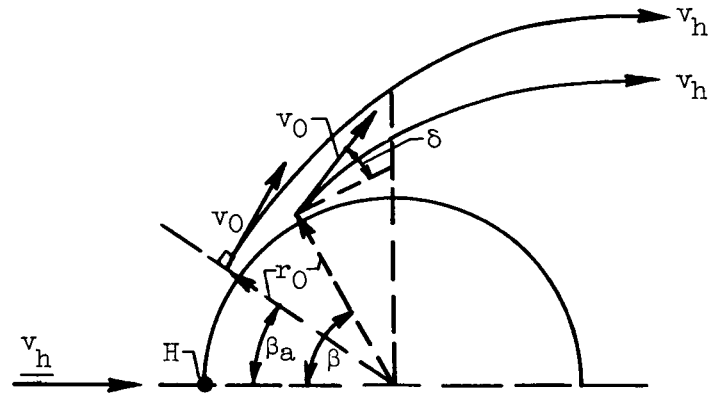
$$\frac{\Delta v}{v} = 2 \sin \frac{i}{2} \cos \nu \approx \sin i \cos \nu \qquad (10)$$

where ν is the angle of approach relative to the normal to the nodal line. Since ν is not far from zero for most trajectories, and v is of the order of 20 miles per second, Δv is of the order of 1/2 to over 1 mile per second for Venus and Mars trajectories. In general, it is necessary to calculate the Δv required at many points along the trajectory to determine when a midcourse correction is minimized. Further consideration of such methods is beyond the scope of this report.

Earth-Launch Condition to Attain a Specified Hyperbolic Velocity

The remainder of the analysis is concerned with determining the launching velocity required at any launching site to attain a desired hyperbolic velocity. To this end, it is necessary first to determine burnout velocity vector v_0 in terms of parameters of the hyperbolic departure trajectories. These burnout parameters can then be related to the terrestrial latitude and hour angle at the launching site. The ultimate objective is to find the actual launching velocity v_L, which must be provided by the launching motors, as function of launch location and time. This launching velocity is the vector difference between the required burnout velocity and the local component of the Earth's rotational speed.

Hyperbolic departure trajectory. - Sketch (e) shows the relation between burnout velocity v_0 and hyperbolic velocity v_h in any plane containing the diameter of the Earth which is parallel to v_h. The



(e)

angle β is the great-circle angle of the launch point from point H, which is the intersection of the diameter parallel to v_h with the Earth's surface. The energy equation,

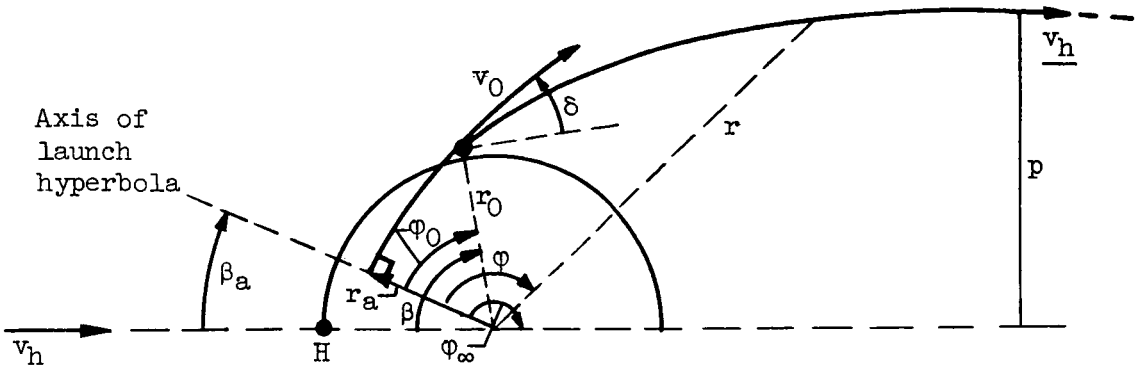
$$v_0^2 - 2\frac{\mu}{r_0} = v_h^2 \tag{11}$$

shows that the final burnout velocity v_0 is independent of β if it always occurs at the same radius r_0. Only the inclination δ relative to the horizontal changes. A value β_a of β exists for which δ is zero. For β less than β_a, the inclination δ is negative; this is generally unsuitable for values of r_0 not much greater than the Earth's radius.

To determine δ as a function of β, the general equation for a conic-section trajectory is utilized:

$$r = \frac{h^2/\mu}{1 + \epsilon \cos \varphi} \tag{12}$$

where h is angular momentum, μ the gravitational constant for the Earth, ϵ the eccentricity, and φ the trajectory angle (true anomaly) measured from the axis of the hyperbola (sketch (f)).



(f)

It is easily shown (e.g., ref. 3) that

$$\epsilon = \frac{v_a^2}{\mu/r_a} - 1 = \frac{v_a^2}{v_{c,a}^2} - 1 \tag{13}$$

where r_a, $v_{c,a}$, and v_a are the radius, circular velocity, and actual velocity at the axis of the hyperbola ($\varphi = 0$). Consequently, from equations (11) to (13),

$$\frac{v_{c,a}^2}{v_{c,0}^2} \equiv \frac{r_0}{r_a} = \frac{v_h^2 + 2v_{c,a}^2}{v_{c,a}^2 + (v_h^2 + v_{c,a}^2)\cos\varphi_0} \tag{14}$$

The inclination δ is given by (ref. 3)

$$\tan\delta = \frac{v_{c,a}}{v_a}\sqrt{\left(\frac{r_0}{r_a}\right)^2\left(\frac{v_a^2}{v_{c,a}^2} - 2\right) + 2\frac{r_0}{r_a} - \frac{v_a^2}{v_{c,a}^2}}$$

$$= \sqrt{\left(\frac{v_{c,a}^2}{v_{c,0}^2} - 1\right)\left[1 + \frac{v_h^2/v_{c,0}^2}{(v_h^2/v_{c,a}^2) + 2}\right]} \tag{15}$$

To determine δ as function of β, note that (sketch (f))

$$\beta + \varphi_\infty = \varphi_0 + 180^\circ \tag{16}$$

where φ_∞ is the value of φ when $r \to \infty$. From equations (12) and (13), this value is

$$\cos \varphi_\infty = -\frac{1}{\epsilon} = -\frac{1}{(v_h^2/v_{c,a}^2) + 1} \tag{17}$$

Solving equations (14) and (16) for $\cos \varphi_0$ gives:

$$-\cos (\beta + \varphi_\infty) = \cos \varphi_0 = \frac{(v_h^2/v_{c,a}) + 2 - (v_{c,a}^2/v_{c,0}^2)}{(v_h^2/v_{c,0}^2) + (v_{c,a}^2/v_{c,0}^2)} \tag{18}$$

For a given $v_{c,0}^2$ (or r_0) and v_h, β is obtained as function of δ with equations (15), (17), and (18) by varying the parameter $v_{c,a}$. The expression for β_a is obtained by setting $\varphi_0 = 0$ in equation (16):

$$\cos \beta_a = \cos (180 - \varphi_\infty) = -\cos \varphi_\infty = \frac{1}{(v_h^2/v_{c,a}^2) + 1} \tag{19}$$

The perpendicular distance p (sketch (f)) of the asymptote from the Earth's center is obtained from conservation of angular momentum.

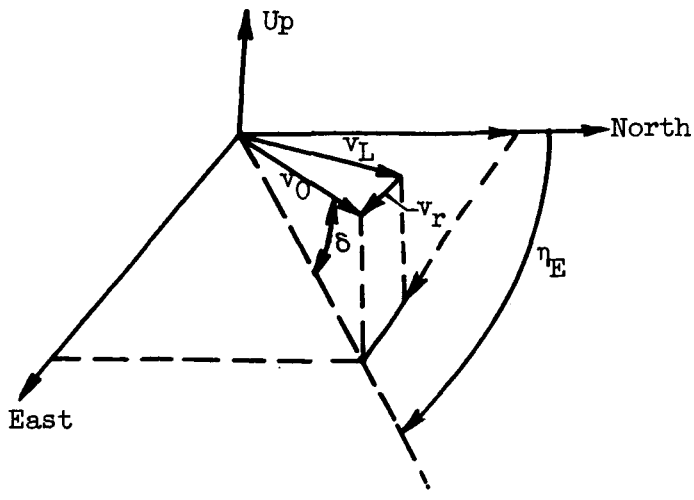
$$pv_h = r_a v_a$$

or

$$\frac{p}{r_a} = \sqrt{1 + 2\frac{v_{c,a}^2}{v_h^2}}$$

This equation shows that the asymptote is normally within a few Earth radii from the extended diameter parallel to $\underline{v_h}$. Thus, if $\underline{v_h}$ and $\underline{v_E}$ (sketch (a)) are not too far from parallel, the vehicle will not be far from the Earth's orbit when hyperbolic velocity is attained.

Determination of magnitude and direction of launching velocity $\underline{v_L}$. - The relation between launching velocity $\underline{v_L}$ and burnout velocity $\underline{v_0}$ is shown in sketch (g). Both $\underline{v_L}$ and $\underline{v_0}$ are based on the burnout radius r_0. The three components of $\underline{v_L}$ can be written as:



(g)

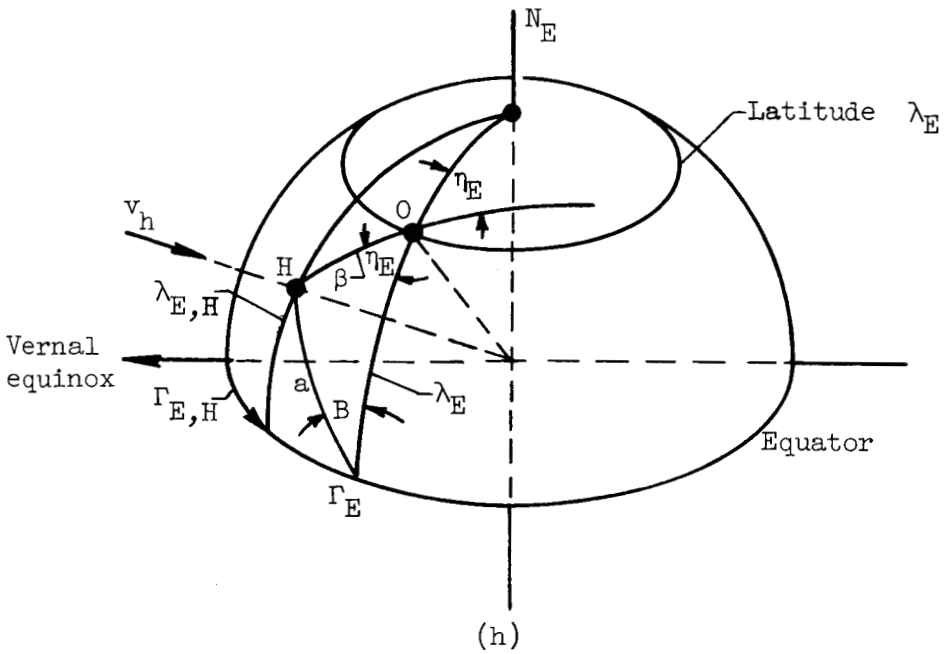
$$\left.\begin{aligned} v_{L,up} &= v_0 \sin \delta \\ v_{L,north} &= v_0 \cos \delta \cos \eta_E \\ v_{L,east} &= v_0 \cos \delta \sin \eta_E - v_r \end{aligned}\right\} \qquad (20)$$

where v_r is the speed due to the Earth's rotation at the launch latitude λ_E, and η_E is the azimuth of the burnout velocity measured clockwise from North. Thus, with $v_r = v_{r,0} \cos \lambda_E$,

$$v_L^2 = v_0^2 - 2v_0 v_{r,0} \cos \delta \sin \eta_E \cos \lambda_E + v_{r,0}^2 \cos^2 \lambda_E \qquad (21)$$

where $v_{r,0}$, the rotational speed of the Earth at the equator, is 0.288 miles per second. If v_L is to be determined as function of launch time and latitude, equation (21) shows that δ and η_E must be expressed in terms of those parameters.

Sketch (h) shows the surface of the sphere of radius r_0 (the burnout radius) with the launch point 0 and the point H of sketches (e) and (f).



The coordinates of the launch point are λ_E and Γ_E, where Γ_E is the longitude of the launch point at launch time and is measured in the direction of the Earth's rotation from the vernal equinox. This longitude is not the fixed geographic longitude; it rotates with the launch site through 360° each day. The hour angle, measured from local apparent noon, is given in terms of Γ_E by

$$\gamma = \frac{\Gamma_E - \Gamma_{E,S}}{15}, \text{ hr} \tag{22}$$

where $\Gamma_{E,S}$ is the longitude of the Sun on the launch date.

Since δ already has been determined as function of β, it is sufficient to express β and η_E (rather than δ and η_E) in terms of λ_E and Γ_E. From the cosine law of spherical trigonometry,

$$\cos \beta = \sin \lambda_E \sin \lambda_{E,H} + \cos \lambda_E \cos \lambda_{E,H} \cos (\Gamma_E - \Gamma_{E,H}) \tag{23}$$

where $\lambda_{E,H}$ and $\Gamma_{E,H}$ are the latitude and longitude of point H.

The azimuth of the burnout velocity η_E is given by

$$\sin \eta_E = \frac{\sin a \sin B}{\sin \beta} \tag{24}$$

where a and B, by the cosine law, are

$$\cos a = \cos \lambda_{E,H} \cos (\Gamma_E - \Gamma_{E,H}) \tag{25}$$

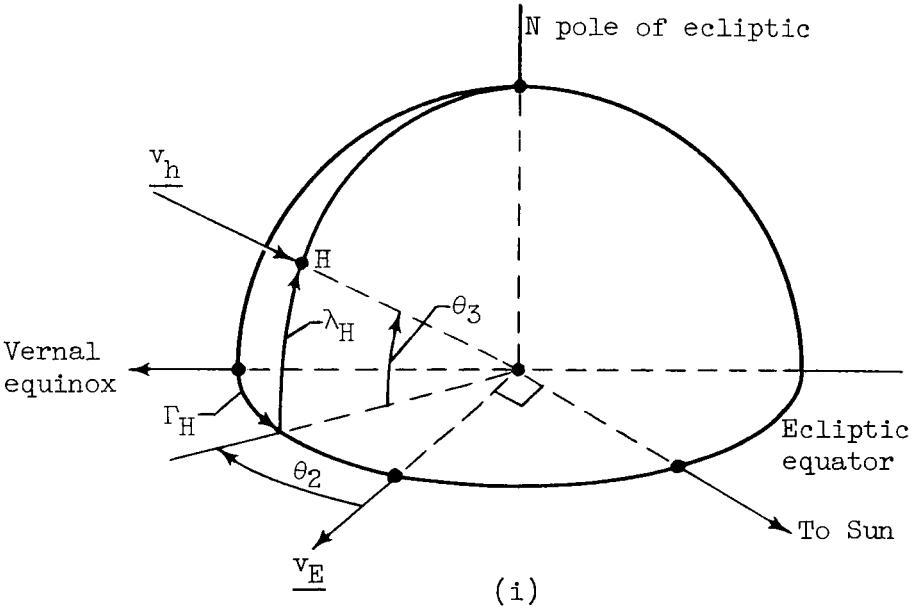
$$\cos B = \frac{\sin \lambda_{E,H}}{\sin a} \tag{26}$$

The resulting equation for η_E is

$$\sin \eta_E = \frac{\cos \lambda_{E,H} \sin(\Gamma_E - \Gamma_{E,H})}{\sin \beta} \tag{27}$$

Equations (23) and (27) are the required expressions for β and η_E in terms of the launch-point coordinates and the coordinates of point H. The latter are best obtained first in terms of ecliptic coordinates, since the direction angles of $\underline{v_h}$ are given in terms of the ecliptic plane. A coordinate transformation then yields the required expressions for $\lambda_{E,H}$ and $\Gamma_{E,H}$ in terms of the direction angles of $\underline{v_h}$.

Ecliptic coordinates of point H. - Sketch (i) shows the coordinates of point H in the ecliptic coordinate system, in which the intersection of the ecliptic plane with the Earth's surface is the fundamental circle (ecliptic equator). Ecliptic longitude Γ is again measured from the



(i)

vernal equinox. It is apparent, by reference to sketch (a), that the latitude and longitude of point H in this system are

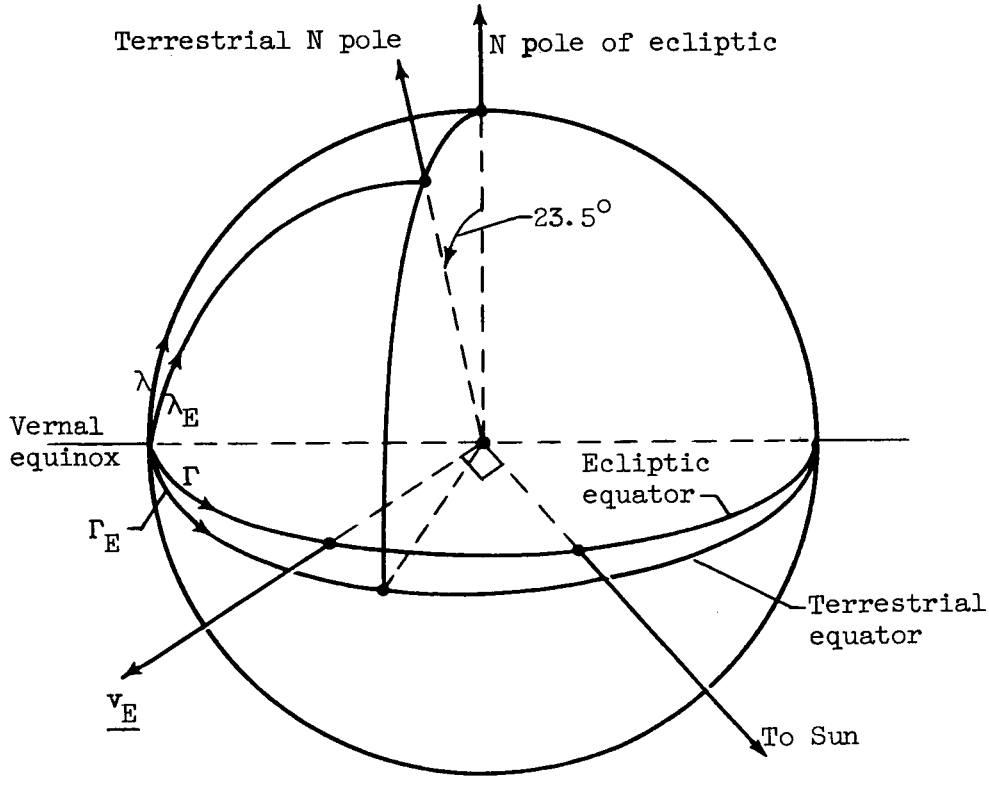
$$\left.\begin{aligned} \lambda_H &= -\theta_3 \\ \Gamma_H &= \Gamma_{v_E} - \theta_2 \end{aligned}\right\} \qquad (28)$$

Thus, a positive λ_H results from a southward inclination of $\underline{v_h}$, and inward (toward the Sun) rotation of v_h reduces Γ_H. The longitude of the Earth's orbital velocity vector is given by

$$\Gamma_{v_E} = \Gamma_S - 90^\circ = \frac{360}{365}(t_L - t_V) - 90^\circ \qquad (29)$$

where Γ_S is the ecliptic longitude of the Sun, t_V is the time of the vernal equinox (March 21), and t_L is the launch date with the time measured in days.

The relation between ecliptic latitude and longitude (λ,Γ) and terrestrial latitude and longitude (λ_E,Γ_E) is shown in sketch (j).



(j)

The transformation equations are (ref. 5)

$$\cos \lambda_E = \frac{\cos \lambda \cos \Gamma}{\cos \Gamma_E} \tag{30}$$

$$\tan \Gamma_E = \frac{-\sin 23.5^\circ \tan \lambda}{\cos \Gamma} + \cos 23.5^\circ \tan \Gamma \tag{31}$$

Thus, the terrestrial coordinates of point H become

$$\tan \Gamma_{E,H} = \frac{\sin 23.5^\circ \tan \theta_3}{\cos(\Gamma_{v_E} - \theta_2)} + \cos 23.5^\circ \tan(\Gamma_{v_E} - \theta_2) \tag{32}$$

$$\cos \lambda_{E,H} = \frac{\cos \theta_3 \cos (\Gamma_{v_E} - \theta_2)}{\cos \Gamma_{E,H}} \tag{33}$$

The terrestrial longitude of the Sun is

$$\tan \Gamma_{E,S} = \cos 23.5^\circ \tan \Gamma_S \tag{34}$$

Summary of procedure for determining v_L. - When the launch date and Earth-planet trajectory have been chosen, the magnitude and direction of v_h are known. The ecliptic longitude of v_E and the Sun are obtained from equation (29) for the launch date. The coordinates of point H are then calculated from equations (32) and (33). The angles β and η_E are obtained from equations (23) and (27) as functions of Γ_E for given λ_E, or as functions of the hour angle γ through equations (22) and (34). The inclination δ is determined as function of β with equations (15), (17), and (18) by varying the parameter $v_{c,a}$. With δ and η_E known as functions of time during the launch day, v_L can be calculated from equation (21) to determine the best launch time.

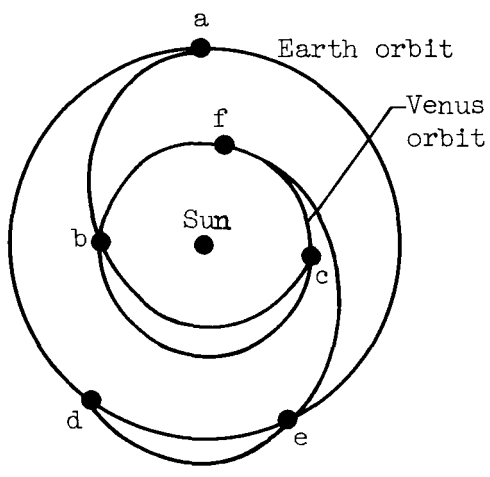
Equation (21) shows that launching can take place from any latitude with a maximum velocity penalty approximately equal to loss of benefit of the Earth's rotational speed. The maximum benefit of this rotational speed occurs approximately when $\cos \delta \sin \eta_E \cos \lambda_E$ is maximized if launch latitude is arbitrary, or when $\cos \delta \sin \eta_E$ is maximized if launch latitude is fixed. The best launching latitude is therefore determined by the direction of the required hyperbolic velocity and is not necessarily the equator.

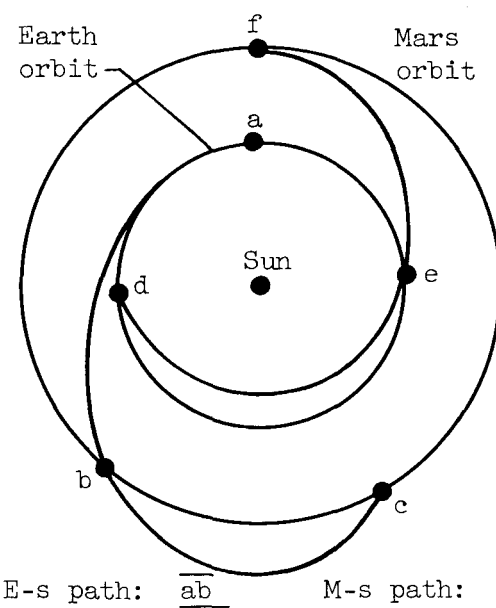
Figure 1 shows the magnitude of the penalty in launching velocity, relative to equatorial launching, for two values of the ratio of hyperbolic

velocity to initial circular velocity and for two values of $\lambda_{E,H}$. Curves for negative $\lambda_{E,H}$ are symmetrical (about $\lambda_E = 0$) with those for positive $\lambda_{E,H}$. It appears from figure 1 that the penalties associated with launching from latitude other than zero are small for a large range of latitude and hyperbolic velocities. For nonzero values of $\lambda_{E,H}$, in fact, nonequatorial launchings show a small advantage over equatorial launchings. For $\lambda_{E,H} = 0$, the velocity penalties shown are only slightly greater than the reduction in the Earth's rotational speed with increasing latitude.

APPLICATION TO EARTH-VENUS TRIPS

A convenient procedure for determining favorable launch periods is to consider the burnout velocity needed as a function of launching date on the basis of coplanar analysis. Many possible trajectories can be taken, but only two families of transfer ellipses are considered herein. Shown in figures 2 and 3 are the coplanar launch velocities needed to reach Venus and Mars, respectively, along these two families. The results were calculated from the data given in reference 3. The four curves correspond to following the long (l) or short (s) branches of ellipses tangent to the Earth's orbit (E) and tangent to the Venus or Mars orbit (V or M) (see sketch (k)). The burnout velocities shown in figures 2 and





E-s path: $\overline{ab}$ V-s path: $\overline{ef}$ E-s path: $\overline{ab}$ M-s path: $\overline{ef}$
E-l path: $\overline{abc}$ V-l path: $\overline{def}$ E-l path: $\overline{abc}$ M-l path: $\overline{def}$

(k)

3 are the coplanar values needed at the assumed launch radius of 1.1 times the radius of the Earth. The orbits of the planets were assumed to be circular, which in the case of Mars can result in errors of about ±0.1 miles per second in v_0. Also shown in figures 2 and 3 are the distance and angle between Earth and planet as a function of time. The departure and arrival patterns repeat themselves each synodic period.

The crossings of the Earth-Venus nodal line are also shown in figure 2 for several synodic periods. The 1959 synodic period is unique in that the Earth crossed the nodal line at minimum-energy departure time (June 7), and Venus crossed the nodal line at arrival (Nov. 2). For this date, therefore, the vehicle could have been launched directly either into the ecliptic plane or into the Venus orbital plane. During the following synodic period, a slightly excess energy trajectory along an E-s ellipse (launching about 2 days after minimum energy) produces an arrival time about May 15, 1961, when Venus is crossing the nodal line (descending node). In general, during each synodic period, there exists a launch date and trajectory that either coincides at departure with an Earth node (June 7 and Dec. 7) or coincides at arrival with a Venus node. Table I shows the range of departure and arrival dates during the 1959 and four succeeding synodic periods, for which v_0 does not exceed 7.00 miles per second (0.2 mile/sec above minimum-energy v_0). In three of these periods (first, second, and fifth), an Earth crossing of the nodal line occurs during the departure period, and in all periods a Venus node occurs during the arrival period. In many cases, however, the required trajectory for these dates may be inconvenient from the standpoints of guidance and communication requirements or duration of the trip; or departure times may be delayed for other reasons. Determination of the amount of excess v_0 required as a function of launching date is therefore desirable to allow for inclination of the Venus orbit. Such computations become rather involved because many families of trajectories should be considered in addition to the two shown in figure 2. For illustrative purposes the computational procedure was carried out for one family of trajectories - namely, the family of ellipses tangent at the Earth's orbit - for the 1959, 1960, and 1962 synodic periods.

Since the coplanar trajectories require resultant heliocentric velocities $\underline{v_1}$ parallel to $\underline{v_E}$ (sketch (a)), the angles α_2 and θ_2 are zero, and $\alpha_1 = \alpha_3 = \alpha$, $\theta_1 = \theta_3 = \theta$, where α is now the inclination of v_1 northward from ecliptic.

Table II shows the velocities and angles calculated for these trajectories as a function of departure days after June 7, 1959. The angle α was calculated from equation (8), v_h from equation (2), θ from equation (3), v_0 from equation (11), and β_a from equation (19). The burnout velocities in this table correspond to a burnout radius of 1.1 times the Earth's radius.

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These values of v_0, together with those for the two succeeding periods, are plotted in figure 4 for comparison. This plot shows that the effect of inclination on v_0 is not large except near the "minimum energy" launch date, if this date does not coincide with a departure or arrival node. A few days after minimum-energy launch time, however, the penalty in v_0 over coplanar values is quite reasonable. The actual minimum-energy launch date, including inclination, is some days after the coplanar minimum-energy date for the trajectories considered. The v_0's are equal to coplanar values when arrival time coincides with a Venus node.

For the 1959 period, table II shows that α and θ jump quite abruptly from zero to a maximum value and then decline as the trajectory plane approaches the ecliptic plane. For a launch date about 90 days after June 7, figure 4 and table II show that α and θ are again zero, since arrival then coincides with another Venus node.

The problem of launching from the Earth's surface at latitude 28° N during a synodic period is illustrated in figures 5 and 6 for two dates during the 1959 period. Figure 5 shows the approximate location of the horizontal-launch circle (β_a) for June 7, and figure 6 for June 27. For the June 7 date, the latitude 28° N lies outside the horizontal-launch circle, so that the resultant velocity v_0 must have some inclination δ at all times during the launch day. To determine the best time of day for the launch, the penalty due to increasing δ must be balanced against the benefit due to increasing η_E. Latitude zero (equator) passes through the center of the horizontal-launch circle, so that full advantage of the Earth's rotation can be realized from an equatorial launch site when $\lambda_H = 0$.

For the June 27 launch (fig. 6), latitude 28° N crosses the horizontal-launch circle at a time when the launch azimuth is not far from east, so that the crossing time would be a favorable launch time. Again, however, some benefit might result from waiting a little to produce even more easterly launch before δ has increased much. Latitude zero passes outside the horizontal-launch circle. Consequently, some velocity penalty would be expected at an equatorial launch site when $\lambda_H > \beta_a$.

The procedure to determine optimum launch time was that outlined in the preceding sections, with $v_{c,0} = 4.69$ for both launch dates and with $v_h = 1.55$ for the June 7 launch and 2.25 for the June 27 launch.

Results of computations for launch from latitude 28° N and from the equator along the E-7 path on June 27 are compared with results for the June 7 minimum-energy trajectory in figure 7. The curves show that, in

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both cases, the launch velocities required from the engines v_L can be very close to those that could be obtained if the full value of the Earth's rotational speed at that latitude were realized. For the June 27 launch, slightly lower launching velocity is required at $\lambda_E = 28^\circ$ than at the equator, while for the June 7 launch the reverse is true. These results for two specific cases agree with the more general conclusions reached from figure 1, and confirm that little penalty in launching velocity is produced by changes in launching latitude, provided that a launch time close to the optimum is selected.

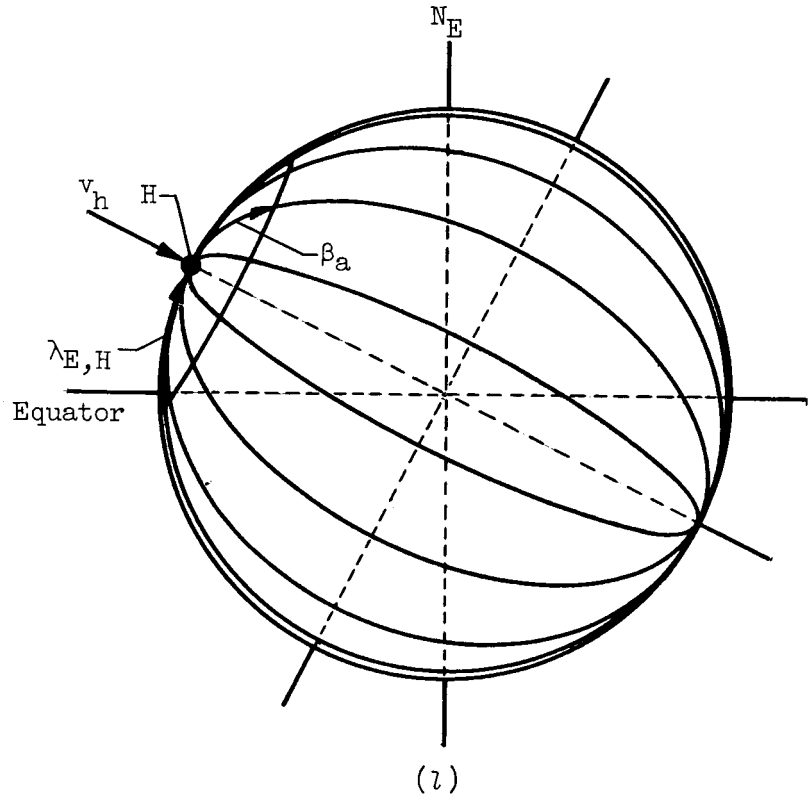
DEPARTURE FROM CIRCULAR SATELLITE ORBITS

The procedure described in the preceding sections for determining launching requirements from the Earth's surface is, of course, directly applicable to launching from the surface of any planet to return to Earth. Some modifications are necessary, however, for trips starting from satellite orbits. In this case the planet's rotation is no longer a factor, but the satellite velocity becomes an even more significant consideration. The problem of launching from circular and elliptic orbits is discussed extensively in reference 4; therefore, only brief consideration is given herein.

Full advantage can be taken of the vehicle's satellite velocity only if the plane of the orbit is parallel to the vector $\underline{v_h}$. Any orbit that passes over the point H has this feature, since all such orbit planes contain the diameter parallel to $\underline{v_h}$. To launch into such a satellite orbit from any terrestrial launching site, the burnout azimuth η_E must be the same function of the hour angle as that required for direct launch from Earth into hyperbolic departure trajectories. In this case, however, the burnout velocity is the circular velocity $v_{c,0}$ at the burnout radius, and the inclination δ is zero for a circular orbit. Consequently, from equation (21), the launch velocity $v_{L,s}$ to attain a circular orbit passing through point H is

$$v_{L,s}^2 = v_{c,0}^2 - 2v_{c,0}v_{r,0} \sin \eta_E \cos \lambda_E + v_{r,0}^2 \cos^2 \lambda_E \qquad (35)$$

Obviously, for a given λ_E, the lowest $v_{L,s}$ is required when η_E is as close to 90° as possible. Sketch (l), which shows the family of great circles (or circular orbits) passing through the point H, illustrates that, for all latitudes more northerly than $\lambda_{E,H}$ or more southerly than $-\lambda_{E,H}$, direct eastward launching into an orbit through H is possible. At other latitudes, η_E cannot be 90°. In particular, at the equator the value of η_E closest to 90° is $\eta_E = 90^\circ + \lambda_{E,H}$. The best launch latitude is approximately that for which $\sin \eta_E \cos \lambda_E$ is maximum.



(l)

Since, from many latitudes, launching into orbit with the full benefit of the Earth's rotational speed is possible, the interplanetary launch from orbit has some advantage, in terms of total velocity increment required, over launch directly from the surface. The remaining increment required to attain the burnout velocity v_0 can be provided when the vehicle passes the β_a circle (sketch (l)), so that δ can be zero. The total velocity increment is, therefore,

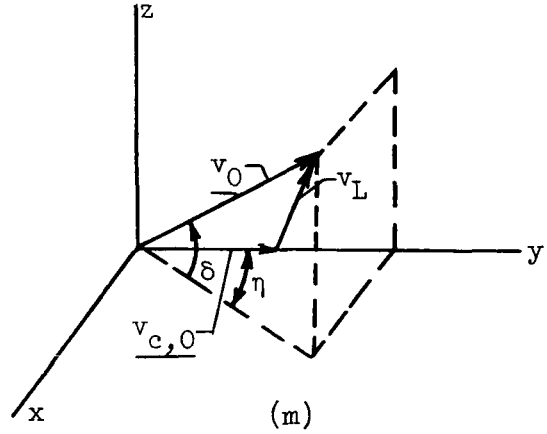
$$v_L = v_{c,0} - v_{r,0} \cos \lambda_E + (v_0 - v_{c,0}) = v_0 - v_{r,0} \cos \lambda_E \qquad (36)$$

which means that the full benefit of the Earth's rotation is obtained. As indicated in the examples of the preceding section, however, this advantage over surface launching is likely to be quite small.

If the vehicle is to be assembled in orbit, the launching time can be selected freely only for the first launching. Subsequent launchings must consider the rendezvous problem. However, by proper selection of orbit period, overhead passage of the vehicle can be produced at about the best launch time on each of several successive days. Some orbit

control or correction would probably be needed, however, to maintain a close correspondence between overhead passage and favorable launching time for extended periods.

If the orbit plane is not parallel to $\underline{v_h}$, quite severe velocity penalties can result. The launch velocity required to produce $\underline{v_0}$ from the satellite velocity $v_{c,0}$ is (sketch (m)).



(m)

$$v_L^2 = (v_0 \cos \delta \cos \eta - v_{c,0})^2 + (v_0 \sin \delta)^2 + (v_0 \cos \delta \sin \eta)^2$$

$$= v_0^2 - 2v_0 v_{c,0} \cos \delta \cos \eta + v_{c,0}^2 \qquad (37)$$

Equation (37) indicates that the penalties associated with large values of δ or η are more severe than for surface launch because $v_{c,0}$ is much greater than the rotational speed of the Earth. For this reason, precession of the orbit plane must be carefully precalculated if the vehicle is to remain in orbit for appreciable periods of time before departure. Furthermore, as pointed out in reference 4, difficulties may arise at the destination planet if the return vehicle remains in orbit. The vehicle may settle into an orbit with inclination as much as 90° relative to the departure direction, so that much of the saving in propellant associated with remaining in orbit, rather than landing, may be negated. Reference 4, however, considers methods whereby the penalty due to deflecting the orbital plane may be reduced by applying the correction impulse after the vehicle has attained large distances from the planet where the vehicle velocity is much reduced.

CONCLUSIONS

The effect of inclination of the orbital plane of the destination planet is to change the required direction and magnitude of the

hyperbolic velocity. The velocity penalty resulting from this inclination is generally quite small - of the order of 0.2 mile per second or less for Earth-Venus trips - unless the departure and arrival points are nearly 180° apart and neither point is at a node. In the latter case, an excess-energy path, in the coplanar sense, may require considerably less velocity than the coplanar minimum-energy path. The velocity penalty associated with orbit inclination is expected to be smaller for a Mars trip than for a Venus trip, because of the smaller inclination of the Mars orbit.

Analysis of the problem of launching interplanetary vehicles from the surface of the Earth indicates that such launchings are possible from any latitude, with the maximum velocity penalty corresponding approximately to loss of the benefit of the Earth's rotational speed. The maximum benefit of this rotational speed is derived when the product $\cos \delta \sin \eta_E \cos \lambda_E$ is maximized, where δ is the upward inclination of the burnout velocity vector, η_E is the azimuth of the burnout velocity, and λ_E is launch latitude. Sample calculations indicate that most of the benefit of the Earth's rotational speed can be realized at any latitude if the launching time during the launch day is properly selected. In some cases, launchings from nonequatorial sites require slightly smaller velocities than equatorial launchings.

Launching interplanetary vehicles from closed orbits around the departure planet may impose much more severe velocity penalties, relative to coplanar values, than launching from the planet's surface, since the satellite velocity is much higher than the surface rotational speed. However, if the selected departure-orbit plane is parallel to the hyperbolic velocity, the total velocity increment may be slightly less than for surface launching. Such departure orbits can be attained from any surface launch site.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, July 15, 1959

APPENDIX - SYMBOLS

A,B,C	angles of spherical triangle
a,b,c	sides of spherical triangle
d	distance from ecliptic plane to orbital plane of destination planet
H	point of intersection with Earth's surface of diameter parallel to v_h
h	angular momentum
i	angle of inclination of orbital plane of destination planet
p	distance of asymptote of launch hyperbola from axis of launch cone
R	distance from Earth to destination planet
R_E	radius of Earth
r	distance to trajectory from mass center
r_a	minimum distance of hyperbola from mass center
r_E	distance from Sun to Earth
r_p	distance from Sun to destination planet
r_0	distance from mass center at which burnout velocity is attained
t_L	launch date
t_V	date of vernal equinox
v	velocity along trajectory
v_a	trajectory velocity at r_a
v_c	circular (satellite) velocity
$v_{c,a}$	circular velocity at r_a
$v_{c,0}$	circular velocity at r_0

v_E	Earth's orbital velocity
v_h	hyperbolic velocity relative to Earth
v_L	launch velocity
$v_{L,s}$	launch velocity to achieve circular satellite orbit
v_r	rotation speed of Earth's surface at launch latitude
$v_{r,0}$	rotation speed of Earth's surface at equator
v_0	burnout velocity at r_0
v_1	heliocentric velocity of vehicle at Earth orbit
α	inclination of $\underline{v_1}$ to $\underline{v_E}$
$\alpha_1, \alpha_2, \alpha_3$	component angles of α (sketch (a))
β	great-circle angle of launch point from point H
β_a	value of β for which v_0 is horizontal
Γ	longitude in ecliptic coordinate system
Γ_E	longitude in terrestrial coordinate system
$\Gamma_{E,H}$	terrestrial longitude of point H
$\Gamma_{E,S}$	terrestrial longitude of Sun
Γ_H	longitude of point H in ecliptic coordinate system
Γ_S	celestial longitude of Sun
Γ_{v_E}	celestial longitude of radius parallel to $\underline{v_E}$
γ	hour angle, relative to local apparent noon
δ	inclination of v_0 from horizontal
ϵ	eccentricity
η	angle between $\underline{v_{c,0}}$ and component of $\underline{v_0}$ in horizontal plane
η_E	azimuth of v_0, counterclockwise from local north

θ	inclination of $\underline{v_h}$ relative to $\underline{v_E}$
θ_M	angle between Mars and Earth
θ_V	angle between Venus and Earth
$\theta_1, \theta_2, \theta_3$	component angles of θ (sketch (a))
λ	latitude in ecliptic coordinate system
λ_E	latitude in terrestrial coordinate system
$\lambda_{E,H}$	latitude in terrestrial coordinate system of point H
λ_H	latitude in ecliptic coordinate system of point H
μ	gravitational constant (9.6×10^4 miles^3/sec^2 for Earth)
ν	angle of intersection of vehicle path with nodal line
φ	trajectory angle, measured from axis of hyperbola
φ_0	value of φ at r_0
φ_∞	value of φ for $r \to \infty$
Ψ_0	angular distance of Earth from ascending node of destination planet
Ψ_1	angular distance of destination planet from its ascending node

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TABLE I. - FAVORABLE DEPARTURE DATES FOR VENUS TRIP

	Departure	Arrival	Departure	Arrival
	Period 1		Period 2	
Minimum energy	June 7, 1959	Nov. 2, 1959	Jan. 13, 1961	June 9, 1961
$v_0 < 7.0$, miles/sec	May 2, 1959 to Aug. 2, 1959	Sept. 20, 1959 to Jan. 22, 1960	Dec. 7, 1960 to Mar. 8, 1961	April 27, 1961 to Aug. 29, 1961
Venus node nearest arrival		Nov. 2, 1959 (ascending)		May 15, 1961 (descending)
	Period 3		Period 4	
Minimum energy	Aug. 19, 1962	Jan. 17, 1963	Mar. 25, 1964	Aug. 23, 1964
$v_0 < 7.0$, miles/sec	July 14, 1962 to Oct. 14, 1962	Dec. 5, 1962 to April 5, 1963	Feb. 20, 1964 to May 20, 1964	July 11, 1964 to Nov. 11, 1964
Venus node nearest arrival		Mar. 20, 1963 (descending)		Oct. 3, 1964 (ascending)
	Period 5			
Minimum energy	Nov. 2, 1965	Mar. 30, 1966		
$v_0 < 7.0$, miles/sec	Sept. 28, 1965 to Dec. 28, 1965	Feb. 18, 1966 to June 18, 1966		
Venus node nearest arrival		April 17, 1966 (descending)		

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TABLE II. - PARAMETERS FOR E-7 VENUS TRIPS

[1959 synodic period.]

Departure days after June 7, 1959	Arrival days after Nov. 2, 1959	v_1, $\frac{\text{mile}}{\text{sec}}$	α, deg	v_h, $\frac{\text{mile}}{\text{sec}}$	θ, deg	Coplanar burnout velocity, v_0, mile/sec	Noncoplanar burnout velocity, v_0, mile/sec	β_a, deg
0	0	16.96	0	1.55	0	6.81	6.81	25.5
1	2.4	16.96	-3.47	1.88	-33.2	6.81	6.89	30.5
5	12	16.91	-4.52	2.11	-39.2	6.83	6.96	33.6
20	41	16.70	-4.30	2.25	-33.9	6.88	7.00	35.5
45	70	16.40	-3.40	2.35	-24.5	6.97	7.04	36.9
60	85	16.19	-2.48	2.42	-16.9	7.02	7.06	37.8
90	112.3	15.75	0	2.75	0	7.18	7.18	41.9

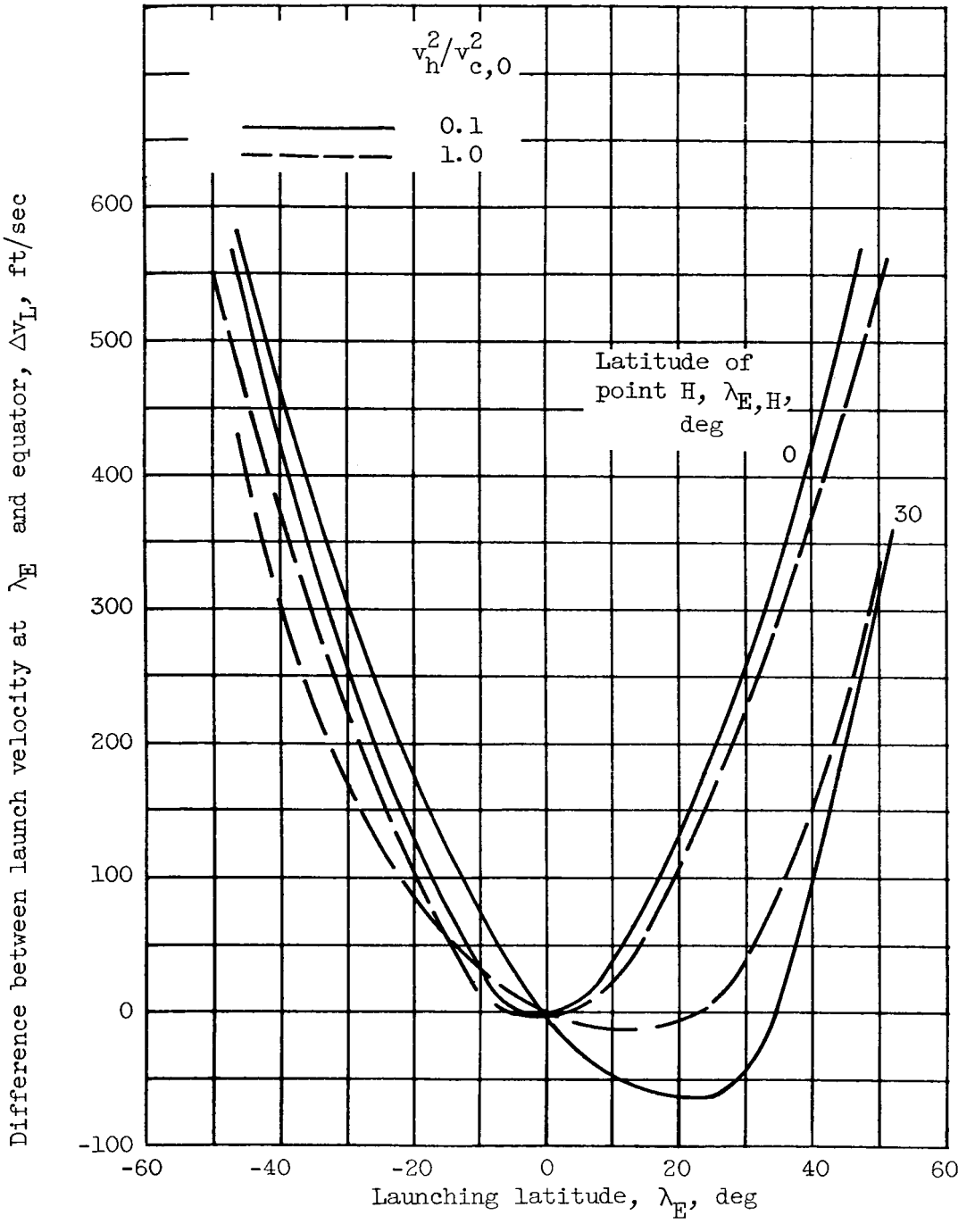

Figure 1. - Effect of launching direction and latitude on launching velocity for $v_{c,0}$ of 4.69 miles per second.

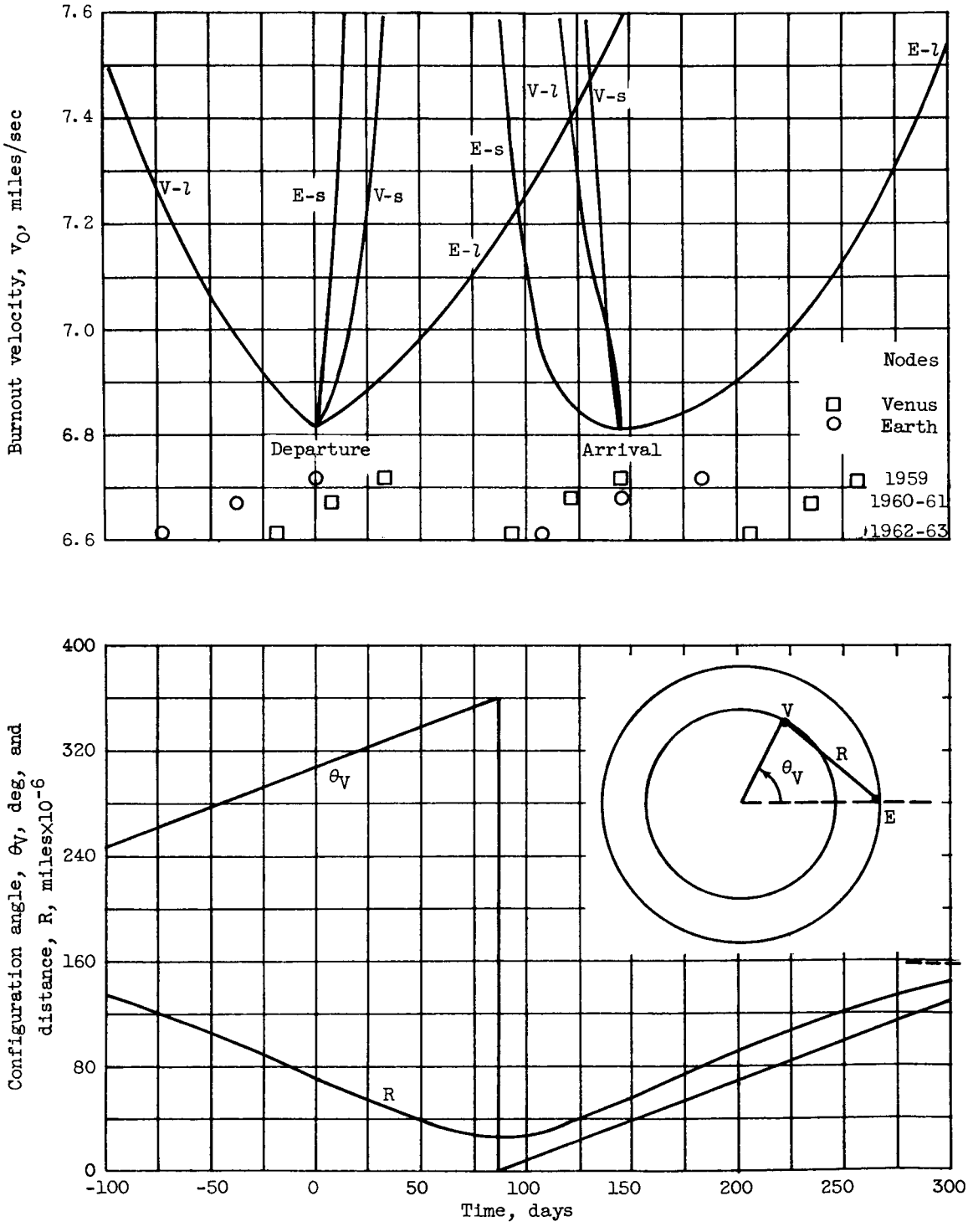

Figure 2. - Coplanar burnout velocity and configuration for Earth-Venus trajectories during synodic period. Burnout radius, 1.1 R_E. For next two periods zero time corresponds to January 13, 1961 and August 19, 1962.

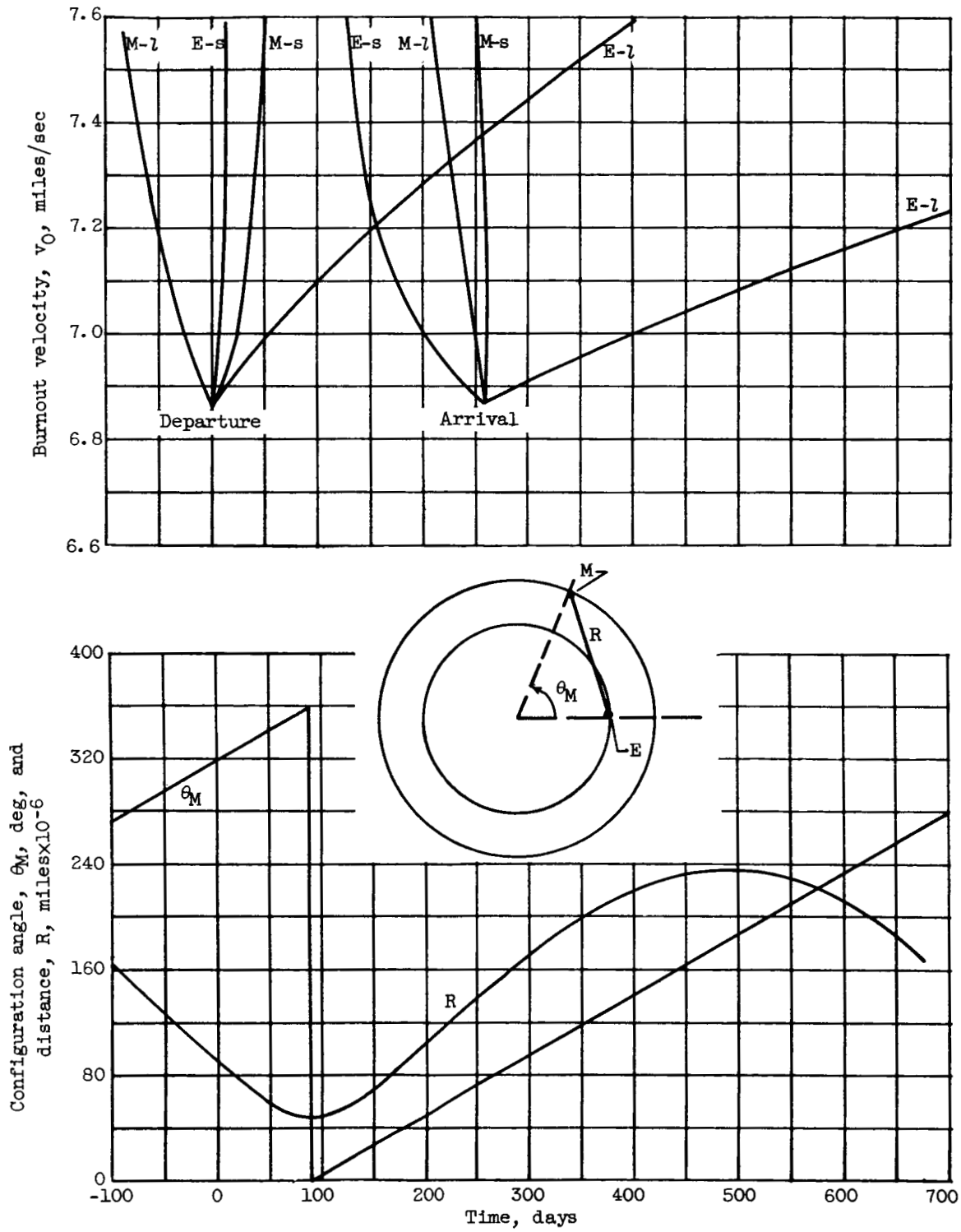

Figure 3. - Coplanar burnout velocities and configuration for Earth-Mars trajectories during synodic period. Burnout radius, 1.1 R_E. For next two periods, zero time corresponds to Oct. 1, 1960 and Nov. 21, 1962.

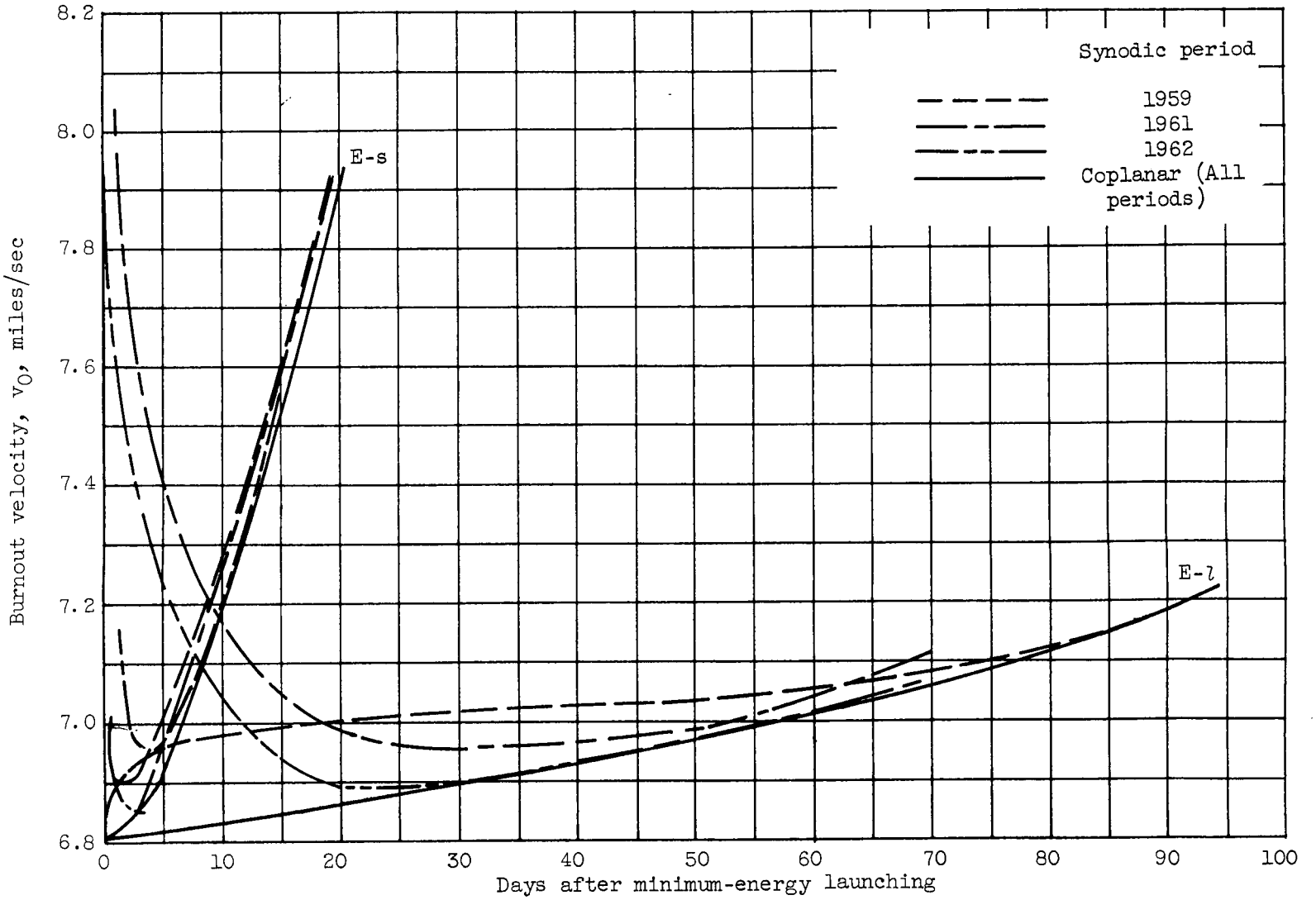

Figure 4. - Effect of inclination of Venus orbit plane on burnout velocity needed.

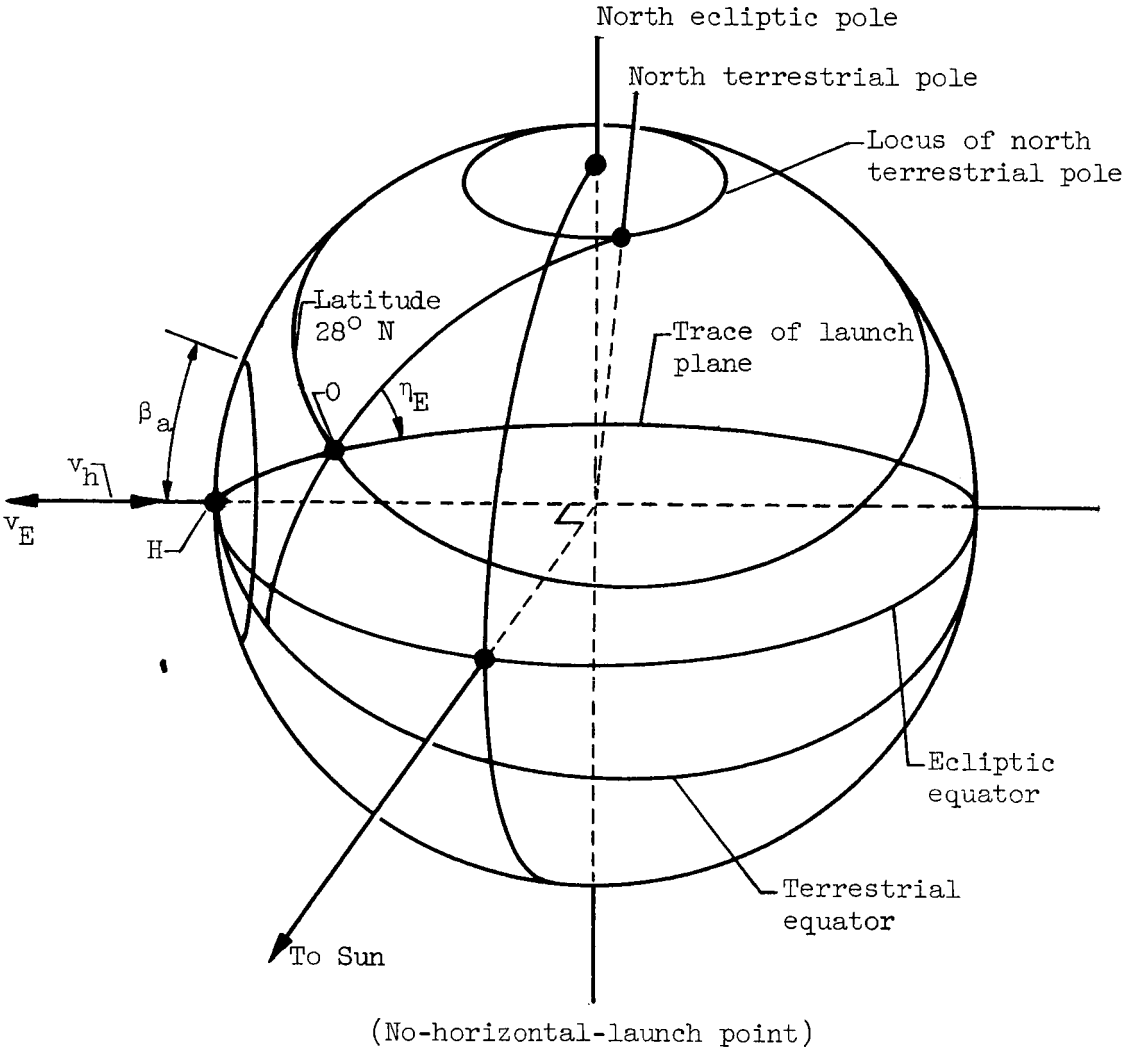

Figure 5. - Launch situation June 7, 1959; $\theta = 0$; $\beta_a = 25.5$.

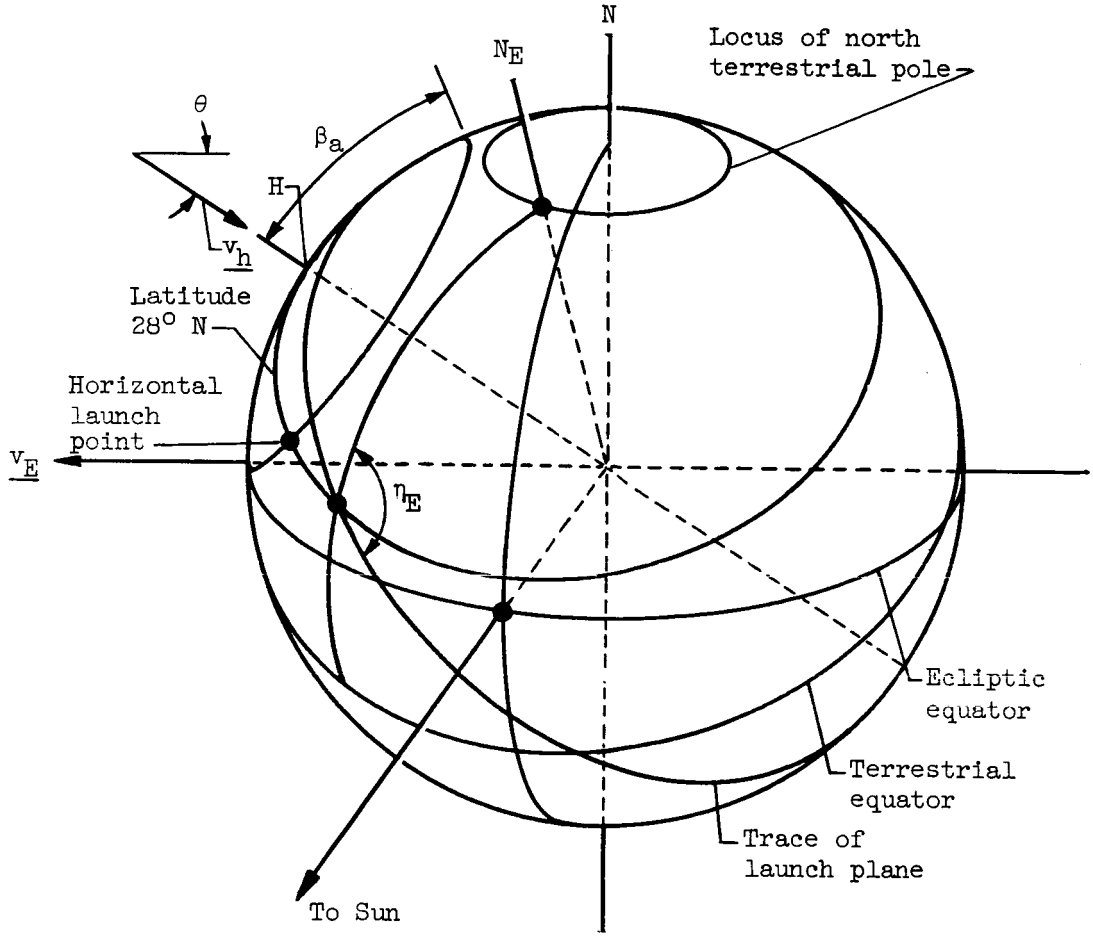

Figure 6. - Launch situation for E-l trajectory about 20 days after June 7, 1959; $\theta = -33.9^{\circ}$; $\beta_a = 35.5$.

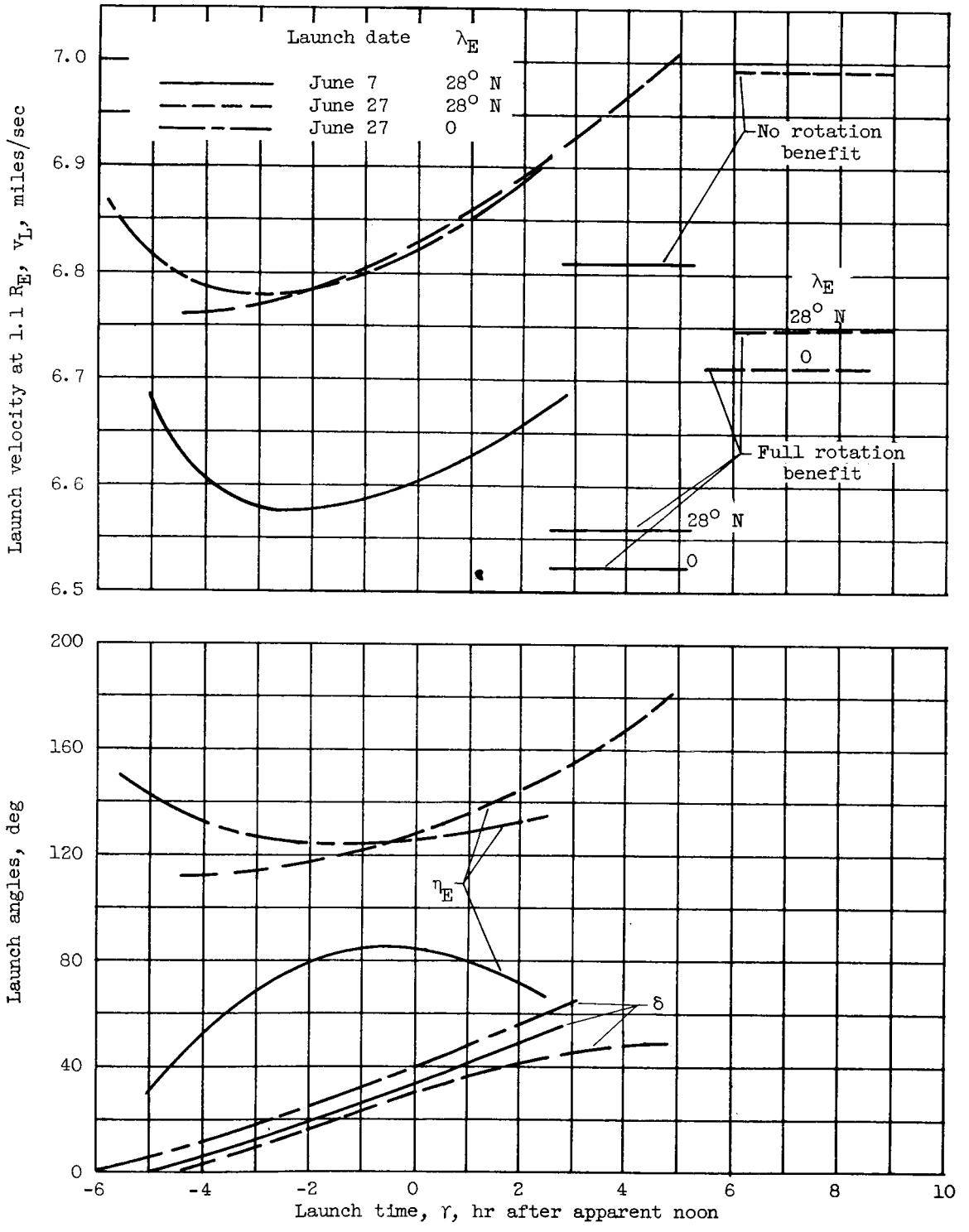

Figure 7. - Launching conditions to reach Venus along E-l path from latitude 28° N during 1959 synodic period.